

Imperfect Interface Model to Study Deformation Field

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Abstract

The properties of bi-materials are significantly influenced by the presence of imperfect interface. Using eigenvalue approach followed by Fourier transformation, the closed form analytical expressions for the deformation field in an isotropic elastic medium consisting of two homogeneous media joined by an imperfect bonding due to inclined line-load has been obtained. In this paper, the imperfect interface has been described by considering a dislocation-like model. The effect of Imperfect interface on the displacement and stresses are studied numerically by considering the different values of constant matrix and observed that the deformation is significantly influenced by the different boundary conditions. The comparisons between perfect and imperfect bonding (dislocation-like) are depicted graphically to demonstrate the significance of imperfect effect at different angles of inclination of line-loading.

Keywords: *Isotropic, Line-loading, Dislocation-like, Eigenvalue, Imperfect Interface.*

1. Introduction

Interface modelling plays an important role in the study of material science and composite structures. To study the properties of macroscopic system physically and mechanically the knowledge of interface is often required. With reference to classical theory of elasticity, an interface establishes itself through interfacial boundary conditions imposed upon equilibrium equations of elasticity. Well defined interfaces are welded interface (perfectly bonded), sliding surface and free surface. It is well known that perfect bonding interface model where the displacement and stresses are continuous across the interface is frequently considered. Practically, when there is de-bonding, sliding (smooth interface)

and/or crossing across interface, the perfect bonding models are not sufficient. The disturbance along a material interface and dislocations significantly influence the physical and mechanical properties of the bi-materials. In early 1950s, the analytical research had started on the interaction between interactions and dislocations by Head [1]. He analyzed screw dislocation near an interface of a biomaterial. The number of static and dynamic problems with imperfect interface has been solved by many researchers e.g. Hashin [2], Yu. [3], Fan & Wang [4] etc. Pan [5] derived the three dimensional Green's functions in anisotropic elastic biomaterials with perfect, smooth, dislocation-like and force-like interfaces. Wu et al. [7] studied elastic field due to dislocation loops in imperfectly bounded isotropic elastic bi-material by considering both the dislocation-like and force-like model.

For a very long source surface, the use of 2-dimensional approximation is justified and makes the calculation simple and one can easily obtained a closed form analytical solution. Chen (2001) studied dislocations interacting with interfaces and inhomogeneities and reviewed the progress of the interaction research in the past thirty years. Schiavone [8] discovered that any classical solution to the boundary value problem is necessarily unique despite the fact that the asymptotic behavior of the solution is not accommodated by the corresponding classical results from the same theory of elasticity. Garg et al. [9] studied the deformation due to inclined line-load present in orthotropic elastic medium with perfect interface. Selim [10] considered the rectangular irregularity in isotropic elastic medium at the interface to study deformation due to normal loading at the origin.

In the present study, the plane strain deformation of an isotropic elastic medium with imperfect interface has been studied. The model considered is dislocation-like interface model due to inclined line-load and the imperfection of interface is described by the constant matrix $K=[k_{ij}]$. When the diagonal elements are unity and non-diagonal elements of constant matrix are zero, the interface becomes perfect and for zero matrix the surface is rigid surface. The effect of constant matrix in the imperfect elastic field are studied numerically for different values of inclination i.e., ($\theta = 0^\circ, 30^\circ, 60^\circ$ & 90°). The results obtained here are important to the study of fracture mechanics and composite mechanics. It is found that deformation field is significantly affected due to imperfect interfaces as compared to perfect bonding.

2. Mathematical Model and its Solution

We have considered an infinite elastic medium consisting of two different imperfectly bounded isotropic elastic homogeneous media which are labelled as M (Medium I, $x_1 > 0$) & M' (Medium II, $x_1 < 0$). Let an inclined line-load L_0 per unit length be act on x_3 -axis and its inclination with x_1 direction is θ .

The problem is two-dimensional plane strain problem in x_1x_2 - plane in which the displacement components u_i in direction of x_i are independent of x_3 co-ordinate so that $u_3 = 0$ and $\frac{\partial}{\partial x_3} \equiv 0$.

As given by Selim [10] the displacement and stresses in M (medium I) and M' (medium II) are given as:

For M

$$u_1^I(x_1, x_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} i \{ [B|k| + C(x_1|k| - T_1^I)] \} e^{|k|x_1} e^{-ikx_2} dk$$

$$u_2^I(x_1, x_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} k \left[\left\{ B + C \left(x_1 - \frac{1}{|k|} \right) \right\} e^{|k|x_1} \right] e^{-ikx_2} dk$$

$$\sigma_1^I(x_1, x_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} i \{ [2Bk^2 + C(2x_1k^2 - |k|T_2^I)] \} e^{|k|x_1} e^{-ikx_2} dk$$

$$\tau_{12}^I(x_1, x_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [\{ 2Bk|k| + C(2x_1k|k| - kT_1^I) \}] e^{|k|x_1} e^{-ikx_2} dk \quad (1)$$

For M'

$$u_1^{II}(x_1, x_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} -i \{ [D|k| + G(x_1|k| + T_1^{II})] \} e^{-|k|x_1} e^{-ikx_2} dk$$

$$u_2^{II}(x_1, x_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} k \left[\left\{ D + G \left(x_1 + \frac{1}{|k|} \right) \right\} e^{-|k|x_1} \right] e^{-ikx_2} dk$$

$$\sigma_1^{II}(x_1, x_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} i \{ [2Dk^2 + G(2x_1k^2 + |k|T_2^{II})] \} e^{-|k|x_1} e^{-ikx_2} dk$$

$$\tau_{12}^{II}(x_1, x_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [\{ 2Dk|k| + G(2x_1k|k| + kT_1^{II}) \}] e^{-|k|x_1} e^{-ikx_2} dk \quad (2)$$

where B, C, D & G are the constants to be determined, k is the Fourier transform parameter,

$$T_1^I = \frac{2\alpha^I}{\alpha^I - 1}, T_1^{II} = \frac{2\alpha^{II}}{\alpha^{II} - 1}$$

$$T_2^I = \frac{4\alpha^I - 2}{\alpha^I - 1}, T_2^{II} = \frac{4\alpha^{II} - 2}{\alpha^{II} - 1}$$

$$\alpha^I = \frac{(\lambda_1 + 2\mu_1)}{\mu_1}, \alpha^{II} = \frac{(\lambda_2 + 2\mu_2)}{\mu_2} \quad (3)$$

and $\lambda_1, \lambda_2, \mu_1$ & μ_2 are Lamé's constants for M and M' respectively, I superscript stands for Medium I & II for Medium II.

2.1. Normal line load

Let a Normal line load L_1 per unit length be acted in the positive x_1 - direction on the imperfect interface $x_1 = 0$ along the x_3 - axis.

Therefore the interface (Imperfect) conditions at $x_1 = 0$ are :

$$u_1^I(0, y) = k_{11}u_1^{II}(0, y) + k_{12}u_2^{II}(0, y)$$

$$u_2^I(0, y) = k_{21}u_1^{II}(0, y) + k_{22}u_2^{II}(0, y)$$

$$\tau_{11}^I(0, x_2) - \tau_{11}^{II}(0, x_2) = -L_1\theta(x_2)$$

$$\tau_{12}^I(0, x_2) - \tau_{12}^{II}(0, x_2) = 0 \quad (4)$$

where k_{ij} ($i, j = 1, 2$) is the constant matrix which describes the bonding conditions along the interface.

Using boundary conditions (4) into the Eqs. (1) & (2), we obtain the following values of constants:

$$\begin{aligned} B &= \frac{L_1}{k^2} F_5 + \frac{L_1}{k|k|} F_6 \\ C &= \frac{L_1}{|k|} F_3 + \frac{L_1}{k} F_4 \\ D &= \frac{L_1}{k^2} F_7 + \frac{L_1}{k|k|} F_8 \\ G &= \frac{L_1}{|k|} F_1 + \frac{L_1}{k} F_2 \end{aligned} \quad (5)$$

where

$$\begin{aligned} F_1 &= \frac{iM_7 - iM_7k_{11} - iM_5 - iM_5k_{22}}{4(M_6M_7 - M_5M_8)}, & F_2 &= \frac{M_7k_{12} - M_5k_{21}}{4(M_6M_7 - M_5M_8)}, \\ F_3 &= \frac{iM_8 - iM_8k_{11} - iM_6 - iM_6k_{22}}{4(M_5M_8 - M_6M_7)}, & F_4 &= \frac{-M_8 - M_6k_{21}}{4(M_5M_8 - M_6M_7)}, \\ F_5 &= \frac{-i}{4} + F_3M_1 + F_1M_2, & F_6 &= F_4M_1 + F_2M_2, & F_7 &= \frac{-i}{4} + F_3M_3 + F_1M_4, & F_8 &= -F_4M_3 - F_2M_4 \\ M_1 &= \frac{T_1^I + T_2^I}{4}, & M_2 &= \frac{T_2^{II} - T_1^{II}}{4}, & M_3 &= \frac{T_2^I - T_1^I}{4}, & M_4 &= \frac{T_1^{II} + T_2^{II}}{4}, \\ M_5 &= M_1 - T_1^I + M_3k_{11} + iM_3k_{12}, \\ M_6 &= M_2 - M_4k_{11} - iM_4k_{12} + k_{11}T_1^{II} + ik_{12}, \\ M_7 &= M_1 - 1 - iM_3k_{21} + M_3k_{22}, \\ M_8 &= M_2 - iM_4k_{21} + M_4k_{22} - k_{21}T_1^{II} - k_{22} \end{aligned} \quad (6)$$

From Eqs. (1) – (5), the deformation field is obtained as:

$$\left. \begin{aligned} u_1^n(x_1, x_2) &= \frac{L_1}{2\pi} \left[\begin{aligned} &i(F_3T_1^I - F_5) \log(x_2^2 + x_1^2) + \\ &2(F_4T_1^I - F_6) \tan^{-1} \left(-\frac{x_2}{x_1} \right) - \\ &\frac{2iF_3x_2^2}{(x_2^2 + x_1^2)} + \frac{2x_1x_2F_4}{(x_2^2 + x_1^2)} \end{aligned} \right] \\ u_2^n(x_1, x_2) &= \frac{L_1}{2\pi} \left[\begin{aligned} &-2iF_5 \tan^{-1} \left(-\frac{x_2}{x_1} \right) + \\ &(F_6 - F_4 - F_5) \log(x_2^2 + x_1^2) \\ &-\frac{2ix_1x_2F_3}{(x_2^2 + x_1^2)} - \frac{2F_4x_1^2}{x_2^2 + x_1^2} \end{aligned} \right] \\ \sigma_1^n(x_1, x_2) &= \frac{L_1}{\pi} \left[\begin{aligned} &\frac{x_1(F_3T_2^I - F_5)}{(x_2^2 + x_1^2)} + \frac{ix_2}{(x_2^2 + x_1^2)} ((F_4T_2^I - F_6) \\ &+ 2F_3x_1 \frac{(x_1^2 - x_2^2)}{(x_2^2 + x_1^2)^2} + \frac{4ix_1^2x_2F_4}{(x_2^2 + x_1^2)^2} \end{aligned} \right] \\ \tau_{12}^n(x_1, x_2) &= \frac{L_1}{\pi} \left[\begin{aligned} &\frac{ix_2}{(x_2^2 + x_1^2)} (F_3T_1^I - 2F_5) + \\ &\frac{x_1}{(x_2^2 + x_1^2)} (F_4T_1^I - 2F_6) - \\ &\frac{4ix_1x_2F_3}{(x_2^2 + x_1^2)^2} + 2F_4 \frac{(x_1^2 - x_2^2)}{(x_2^2 + x_1^2)^2} \end{aligned} \right] \end{aligned} \quad (7)$$

The superscript **n** indicates the deformation due to line load acting normally.

2.2. Tangential line load

Let a tangential line load L_2 per unit length be acted at $x_1 = 0$ in the positive x_2 –direction. Then the Interface (Imperfect) condition at the horizontal plane ($x_1 = 0$) are

$$\begin{aligned} u_1^I(0, y) &= k_{11}u_1^{II}(0, y) + k_{12}u_2^{II}(0, y) \\ u_2^I(0, y) &= k_{21}u_1^{II}(0, y) + k_{22}u_2^{II}(0, y) \\ \tau_{11}^I(0, y) - \tau_{11}^{II}(0, y) &= 0 \\ \tau_{12}^I(0, y) - \tau_{12}^{II}(0, y) &= -L_2\theta(y) \end{aligned} \quad (8)$$

After solving the Eqs. (1) - (9) we have obtained the following values of constants:

$$\begin{aligned} B &= \frac{L_2}{k^2} F_{13} + \frac{L_2}{k|k|} F_{14} \\ C &= \frac{L_2}{|k|} F_{11} + \frac{L_2}{k} F_{12} \\ D &= \frac{L_2}{k^2} F_{15} + \frac{L_2}{k|k|} F_{16} \\ G &= \frac{L_2}{|k|} F_9 + \frac{L_2}{k} F_{10} \end{aligned} \quad (9)$$

Where

$$\begin{aligned} F_9 &= \frac{iM_7 + iM_7k_{11} - iM_5 + M_5k_{22}}{4(M_6M_7 - M_5M_8)}, & F_{10} &= \frac{M_7k_{12} + M_5k_{21}}{4(M_6M_7 - M_5M_8)}, \\ F_{11} &= \frac{iM_8 + iM_8k_{11} - iM_6 + iM_6k_{22}}{4(M_5M_8 - M_6M_7)}, & F_{12} &= \frac{M_8 + M_6k_{21}}{4(M_5M_8 - M_6M_7)}, \\ F_{13} &= \frac{-i}{4} + F_{11}M_1 + F_9M_2, & F_{14} &= F_{12}M_1 - F_{10}M_2, \\ F_{15} &= \frac{-i}{4} + F_{11}M_3 + F_9M_4, & F_{16} &= -F_{12}M_3 - F_{10}M_4 \end{aligned} \quad (10)$$

From Eqs. (1)- (8), we obtain the following analytical expressions for deformation field as:

$$\begin{aligned}
 u_1^t(x_1, x_2) &= \frac{L_2}{2\pi} \left[\begin{aligned} &i(F_{11}T_1^I - F_{13}) \log(x_2^2 + x_1^2) \\ &+ 2(F_{12}T_1^I - F_{14}) \tan^{-1} \left(\frac{x_2}{x_1} \right) - \\ &\frac{2iF_{11}x_1^2}{(x_2^2+x_1^2)} + \frac{2x_1x_2F_{12}}{(x_2^2+x_1^2)} \end{aligned} \right] \\
 u_2^t(x_1, x_2) &= \frac{L_2}{2\pi} \left[\begin{aligned} &-2i(F_{11} - F_{13}) \tan^{-1} \left(\frac{x_2}{x_1} \right) \\ &+ (F_{12} - F_{14}) \log(x_2^2 + x_1^2) \\ &- \frac{2ix_1x_2F_{11}}{(x_2^2+x_1^2)} - \frac{2F_{12}x_1^2}{(x_2^2+x_1^2)} \end{aligned} \right] \\
 \sigma_1^t(x_1, x_2) &= \frac{L_2}{\pi} \left[\begin{aligned} &\frac{x_1(F_{11}T_2^I - F_{13})}{(x_2^2+x_1^2)} + \\ &\frac{ix_2}{(x_2^2+x_1^2)} ((F_{12}T_2^I - F_{14})) \\ &+ 2F_{11}x_1 \frac{(x_1^2-x_2^2)}{(x_2^2+x_1^2)^2} + \frac{4ix_1^2x_2F_{12}}{(x_2^2+x_1^2)^2} \end{aligned} \right] \\
 \tau_{12}^t(x_1, x_2) &= \frac{L_2}{\pi} \left[\begin{aligned} &\frac{ix_2}{(x_2^2+x_1^2)} (F_{11}T_1^I - 2F_{13}) \\ &+ \frac{x_1}{(x_2^2+x_1^2)} (F_{12}T_1^I - 2F_{11}) - \frac{4ix_1x_1F_{11}}{(x_2^2+x_1^2)^2} + \\ &2F_{12} \frac{(x_1^2-x_2^2)}{(x_2^2+x_1^2)^2} \end{aligned} \right]
 \end{aligned} \tag{11}$$

The superscript t indicates the deformation due to line load acting tangentially.

2.3. Inclined line load

For an inclined line load L_0 , we have:

$$L_1 = L_0 \cos \theta, L_2 = L_0 \sin \theta \tag{12}$$

The deformation due to incline line load can be obtained by the superposition of tangential and normal line loading. The final deformation of the considered problem indicates the results for inclined line load :

$$\left. \begin{aligned}
 u_1(x, y) &= u_1^n(x, y) + u_1^t(x, y) \\
 u_2(x, y) &= u_2^n(x, y) + u_2^t(x, y) \\
 \sigma_1(x, y) &= \sigma_1^n(x, y) + \sigma_1^t(x, y) \\
 \tau_{12}(x, y) &= \tau_{12}^n(x, y) + \tau_{12}^t(x, y)
 \end{aligned} \right\} \tag{13}$$

Where deformation due to a normal line load L_1 and a tangential line load L_2 are obtained earlier in section (2.1) & (2.2) respectively.

3. Numerical results and Discussion

To obtain the effect of inclined line-load on displacements and stresses in isotropic elastic half space with imperfect bonding, we use the closed form analytic expression obtained in eq. (13), as a result of inclined. To gain more perspective view of the solution obtained in eq. (13) and the effect of different angles of inclination, we have examined them numerically and graphically by considering following values of Lamé's constants [10].

$$\mu_1 = 1.90930, \quad \mu_2 = 2.14060, \quad \lambda_1 = 2.22075, \\ \lambda_2 = 2.72075,$$

in terms of a unit stress of 10^{11} dyne/cm².

To show the variations in deformation we have considered the following two different constant matrices which describe the imperfection of interface:

$$K1 = [k_{ij}] = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.5 \end{bmatrix}, K2 = [k_{ij}] = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.75 \end{bmatrix}$$

and for perfect interface

$$K3 = [k_{ij}] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

In figures (1) -(4), the variations of normal displacements against the horizontal distance (y) at $x=0.5$ & at $x=1$ with different imperfect matrix parameters. From these figures it has been observed that as the horizontal distance increases, the displacement and distance between the displacements also increases for K1 while for K2 the displacements slightly increase and also decreases continuously.

Variation of tangential displacements at K1 and K2 has been depicted in figures (5)-(8). In figures (5) & (6), it has been observed that at different angles of inclination, initially the displacements in magnitude increases and then decreases for K1 while for K2, the displacements continuously decrease.

Further in figures (9) - (14) the comparison has been done between the displacements due to imperfect interfaces (K1 & K2) and the displacements due to perfect interface (K3) at different values of $x=0.5, 1, 1.25$. From these figures it is found that at $x=1.0$ and $x=1.25$ as the horizontal distance increases the difference between normal displacements due to K1, K2 and K3 also increases while at $x=0.5$ the difference initially decreases and then abruptly increases. Whereas in case of tangential displacements as the horizontal distance increases the difference between the displacements also increases.

In figures (15) - (18), the comparison has been done between the stresses due to imperfect and perfect interfaces. In case of normal stress, the stress slightly increases and then decreases in magnitude and distance between them is also decreases. Figure (17) and (18) shows that at the interface $x=0$ there is a remarkably difference between the tangential stresses due to imperfect interface and perfect interface. From these figures it is observed that near the interface stresses effected remarkably.

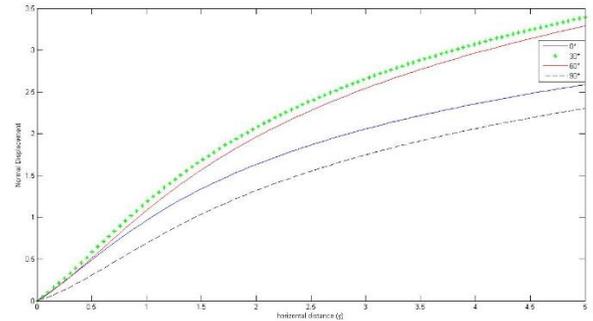


Figure 1: Variation of u_1 (normal displacement) with x_2 (horizontal distance) on the plane at $x=1$ for the Matrix K1 at size of $\delta = 0^\circ, \delta = 30^\circ, \delta = 60^\circ$ & $\delta = 90^\circ$

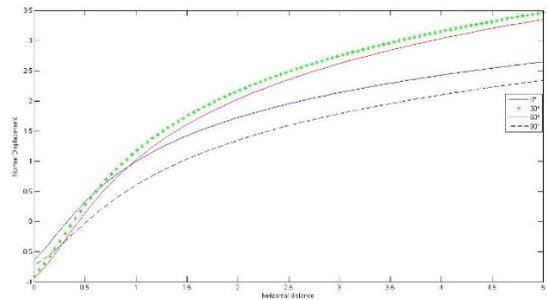


Figure 2: Variation of u_1 (normal displacement) with x_2 (horizontal distance) on the plane at $x=0.5$ for the Matrix K1 at size of $\delta = 0^\circ, \delta = 30^\circ, \delta = 60^\circ$ & $\delta = 90^\circ$

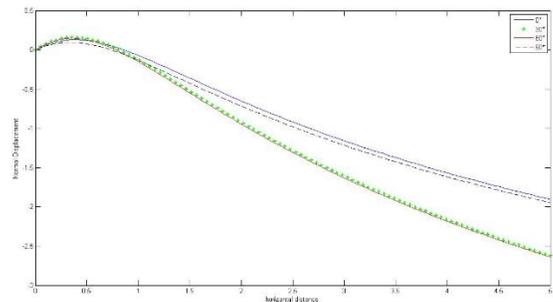


Figure 3: Variation of u_1 (normal displacement) with x_2 (horizontal distance) on the plane at $x=1$ for the Matrix K2 at size of $\delta = 0^\circ, \delta = 30^\circ, \delta = 60^\circ$ & $\delta = 90^\circ$

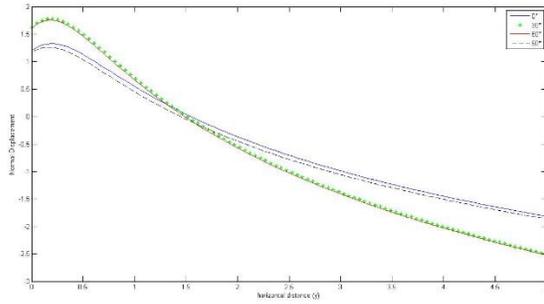


Figure 4: Variation of u_1 (normal displacement) with x_2 (horizontal distance) on the plane at $x=0.5$ for the Matrix K2 at size of $\delta = 0^0, \delta = 30^0, \delta = 60^0$ & $\delta = 90^0$

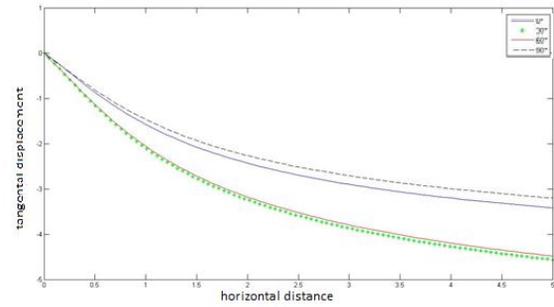


Figure 7: Variation of u_2 (tangential displacement) with x_2 (horizontal distance) on the plane at $x=1$ for the Matrix K2 at size of $\delta = 0^0, \delta = 30^0, \delta = 60^0$ & $\delta = 90^0$

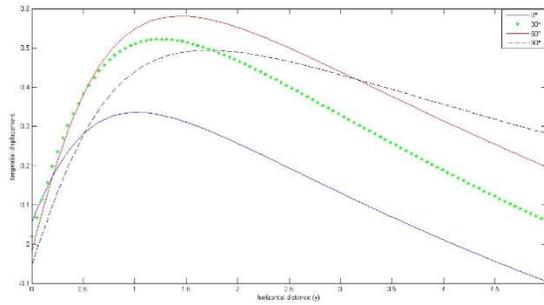


Figure 5: Variation of u_2 (tangential displacement) with x_2 (horizontal distance) on the plane at $x=1$ for the Matrix K1 at size of $\delta = 0^0, \delta = 30^0, \delta = 60^0$ & $\delta = 90^0$

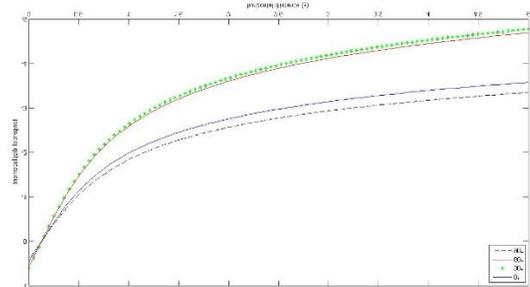


Figure 8: Variation of u_2 (tangential displacement) with x_2 (horizontal distance) on the plane at $x=0.5$ for the Matrix K2 at size of $\delta = 0^0, \delta = 30^0, \delta = 60^0$ & $\delta = 90^0$

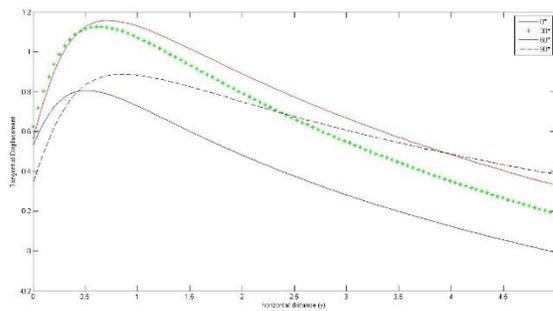


Figure 6: Variation of u_2 (tangential displacement) with x_2 (horizontal distance) on the plane at $x=0.5$ for the Matrix K1 at size of $\delta = 0^0, \delta = 30^0, \delta = 60^0$ & $\delta = 90^0$

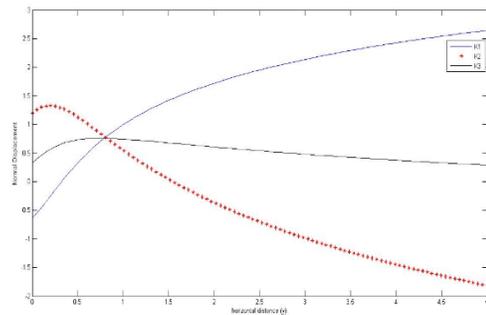


Figure 9: Comparison between the normal displacement due to K1 , K2 and K3 at $x=0.5$

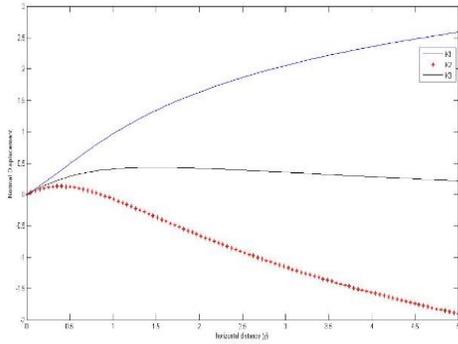


Figure 10: Comparison between the normal displacement due to K1, K2 and K3 at x=1

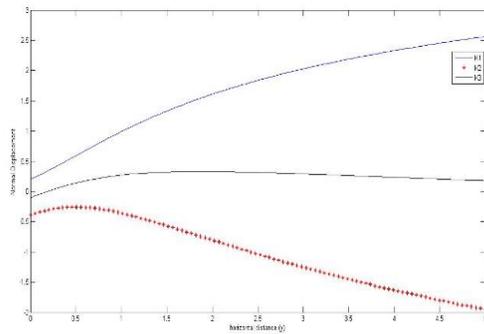


Figure 11: Comparison between the normal displacement due to K1, K2 and K3 at x=1.25

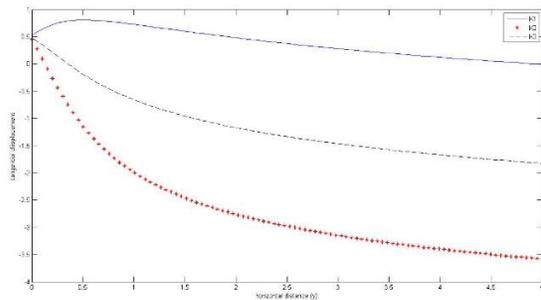


Figure 12: Comparison between the tangential displacement due to K1, K2 and K3 at x=0.5

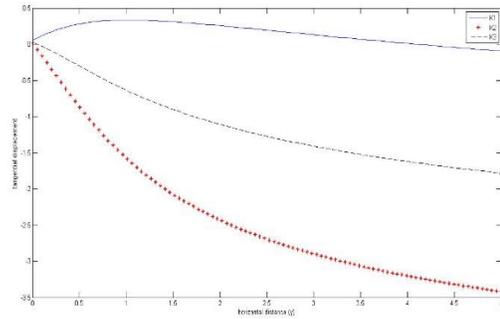


Figure 13: Comparison between the tangential displacement due to K1, K2 and K3 at x=1.0

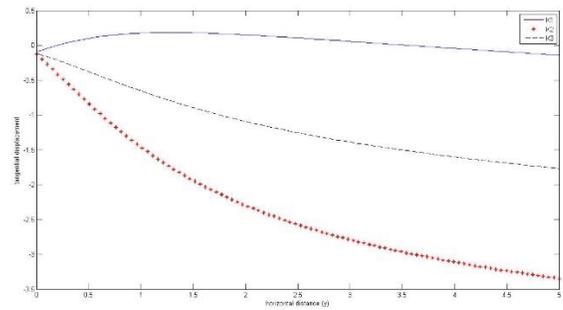


Figure 14: Comparison between the tangential displacement due to K1, K2 and K3 at x=1.25

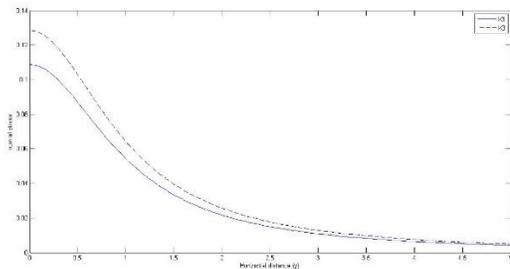


Figure 15: Comparison between normal stresses due to constant matrix K1 and K3 at x=0 for $\delta = 60^\circ$

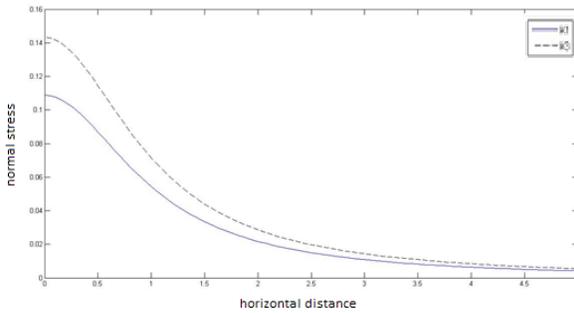


Figure 16: Comparison between normal stresses due to constant matrix K1 and K3 at x=0 for $\delta = 30^\circ$

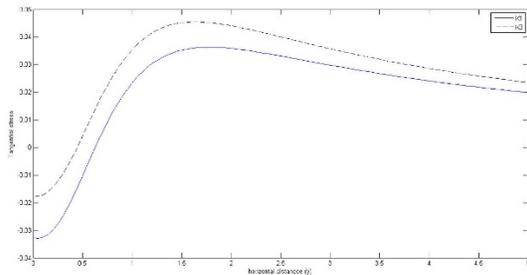


Figure 17: Comparison between tangential stresses due to constant matrix K1 and K3 at x=0 for $\delta = 60^\circ$

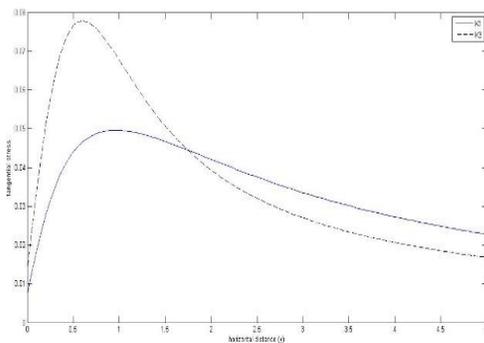


Figure 18: Comparison between tangential stresses due to constant matrix K1 and K3 at x=0 for $\delta = 30^\circ$

Appendix A. ($x > 0$)

$$\int_{-\infty}^{\infty} \exp(-|k|x) \exp(-iky) dk = \frac{2x}{y^2 + x^2}$$

$$\int_{-\infty}^{\infty} \frac{k}{|k|} \exp(-|k|x) \exp(-iky) dk = \frac{-2iy}{y^2 + x^2}$$

$$\int_{-\infty}^{\infty} (|k|)^{-1} \exp(-|k|x) \exp(-iky) dk = -\log(y^2 + x^2)$$

$$\int_{-\infty}^{\infty} |k| \exp(-|k|x) \exp(-iky) dk = \frac{2(x^2 - y^2)}{(y^2 + x^2)^2}$$

$$\int_{-\infty}^{\infty} k \exp(-|k|x) \exp(-iky) dk = \frac{-4iyx}{(y^2 + x^2)^2}$$

$$\int_{-\infty}^{\infty} \frac{1}{k} \exp(-|k|x) \exp(-iky) dk = -2i \tan^{-1} \left(\frac{y}{x} \right)$$

References

- [1] A.K. Head, The interaction of dislocation and boundaries. *Phil. Mag.* Vol. 44, 1953, pp. 92-94.
- [2] Z. Hashin, The spherial inclusion with imperfect interface. *J. Appl. Mech.* Vol. 58, 1991, pp. 444-449.
- [3] HY. Yu, A new dislocation-like model for imperfect interfaces and their effect on load transfer. *Compos Part A*, Vol. 28, 1998, pp. 1057-1062.
- [4] H. Fan, G.F. Wang, Screw dislocation interacting with imperfect interface. *Mech. Materials*. Vol. 35, 2003, pp.943-953.
- [5] E. Pan, Three dimensional interfacial Green's function in anisotropic bimetals. *Applied Mathematical Modeling*. Vol. 27, 2003, pp. 307-326.
- [6] B.J. Chen, Dislocation theory and its application in fracture analysis. Ph. D thesis, School of Mechanical and Production Engineering, Nanyang Technological University, Singapore 2001.
- [7] P. Schiavone, Uniqueness in inclusion problems with imperfect interface. Vol. 53, 2015, pp. 255-257.
- [8] Wu Wenwang, Lv. Cunjing, Xu Shucui, Zhang Jinhuan, Elastic field due to dislocation loops in isotropic bimetals with dislocation-like and

force-like interface models. Mathematics and Mechanics of Solids ,2016, pp.1-15.

- [9] N.R. Garg, R. Kumar, A. Goel, A. Miglani, Plain strain deformation of an orthotropic elastic medium using an eigenvalue approach. Earth Planet Sci., Vol. 55, 2003, pp. 3-9.
- [10] M.M. Selim, Effect of irregularity on static deformation of elastic half-space. International J.Modern Phys. Vol. 22, No. 14, 2008, pp. 2241-2253.